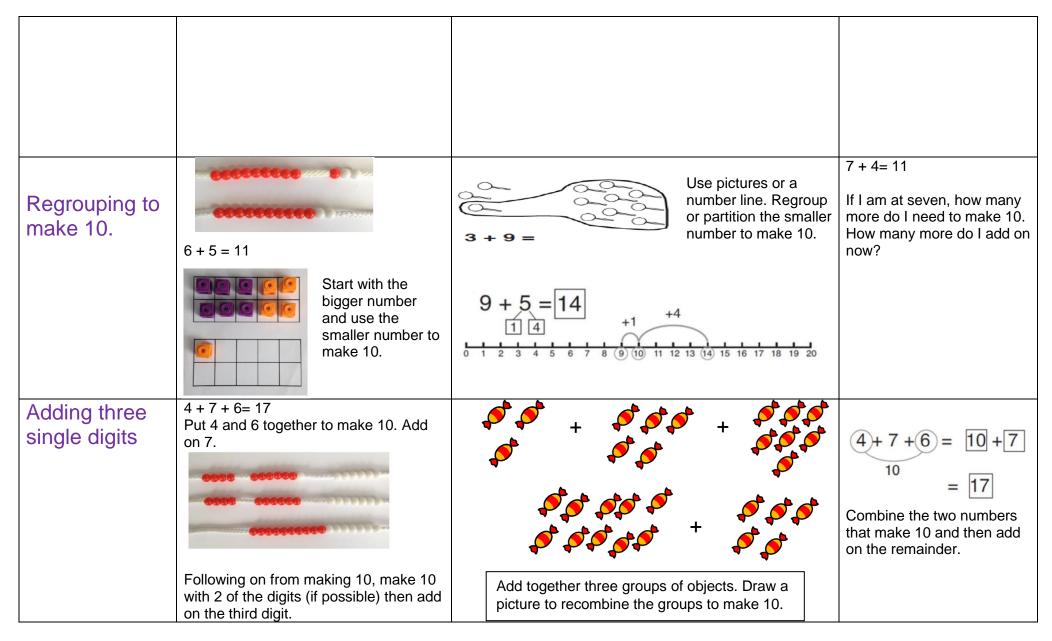
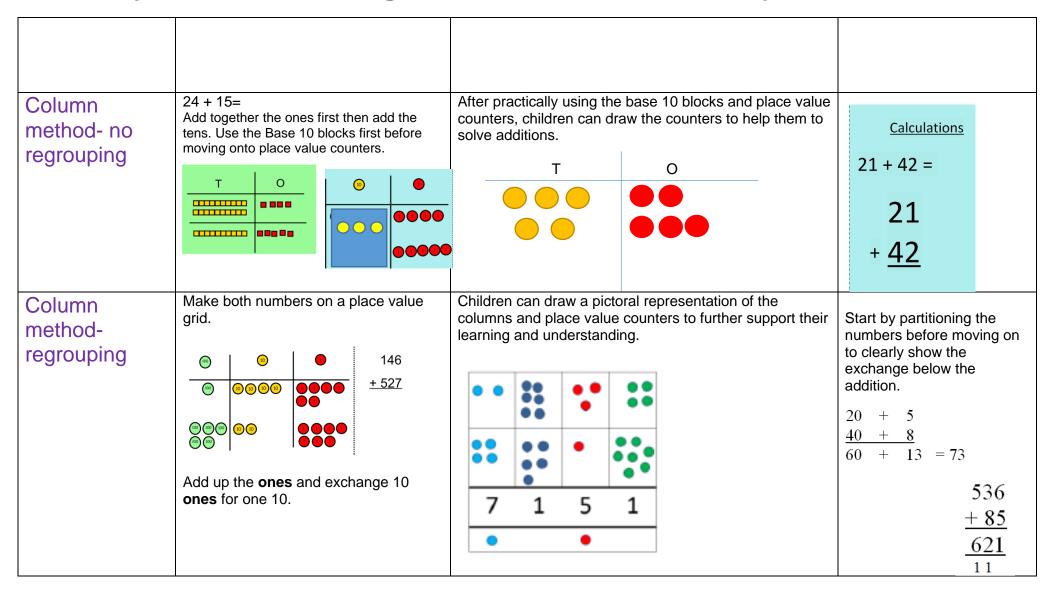
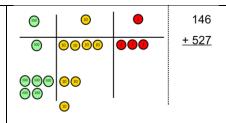
Progression in Calculations

Addition

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	Use cubes to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7 10= 6 + 4 5 Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting	, CCCCCCCCCC	12 + 5 = 17	5 + 12 = 17
on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	10 11 12 13 14 15 16 17 18 19 20	
		Start at the larger number on the number line and count on in ones or in one jump to find the answer.	Place the larger number in your head and count on the smaller number to find your answer.







Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

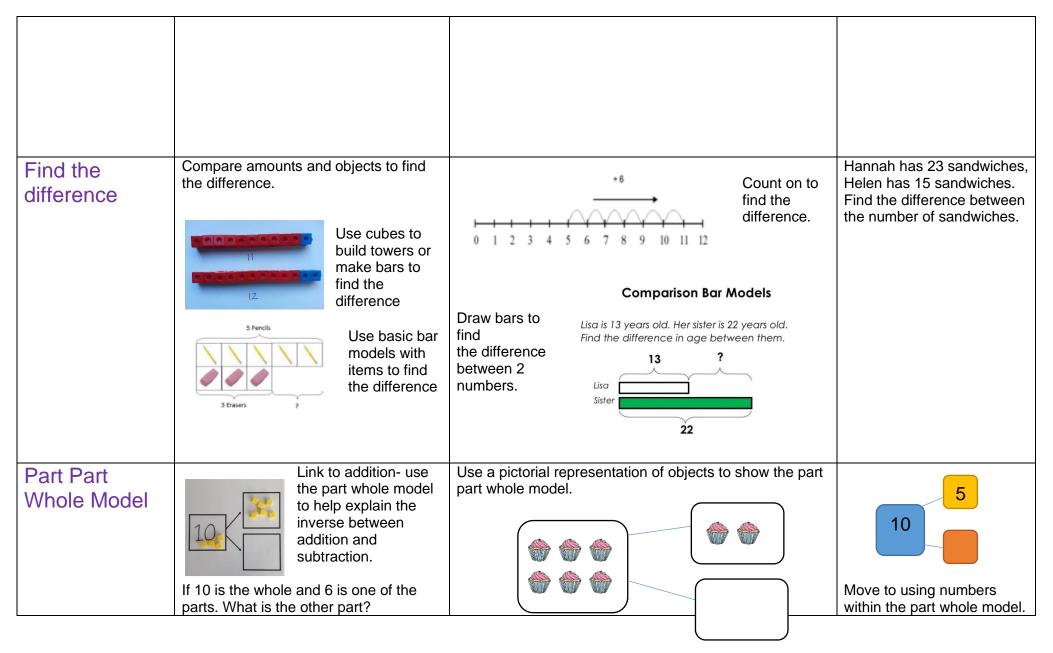
This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

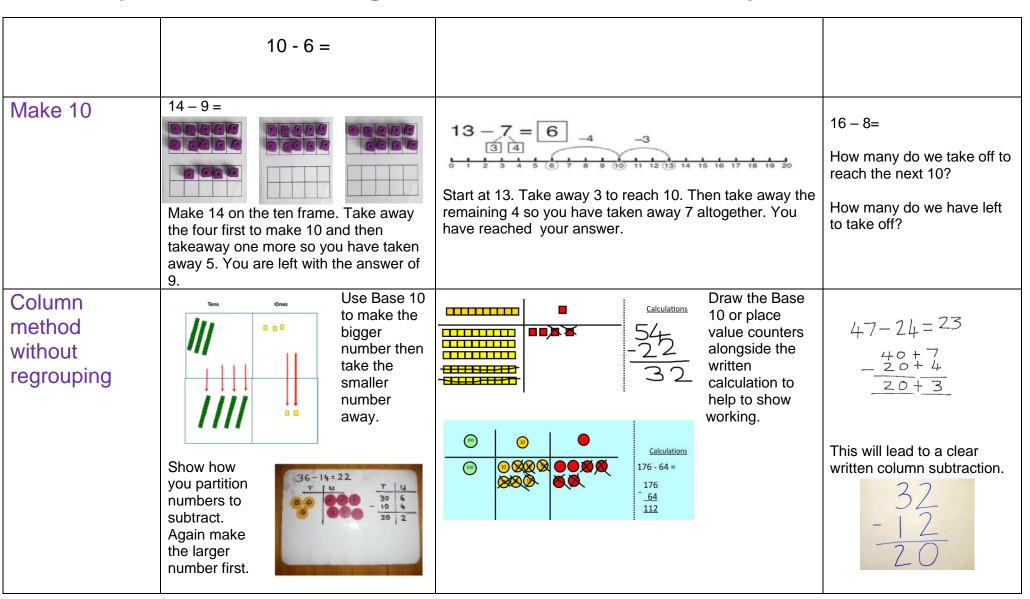
As children move on to decimals, money and decimal place value counters can be used to support learning.

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

Subtraction

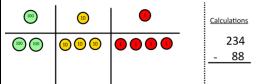
Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away. $6-2=4$	Cross out drawn objects to show what has been taken away.	18 -3= 15 8 - 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.	Count back on a number line or number track 9 10 11 12 13 14 15 Start at the bigger number and count back the smaller	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
	Use counters and move them away from the group as you take them away counting backwards as you go.	number showing the jumps on the number line. -10 -10 -10 -10 34 35 36 37 47 This can progress all the way to counting back using two 2 digit numbers.	



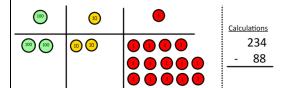


Column method with regrouping Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters

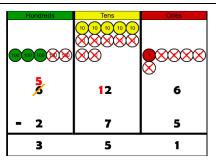


Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

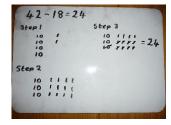


Now I can subtract my ones.

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method

and knows when to exchange/regroup.

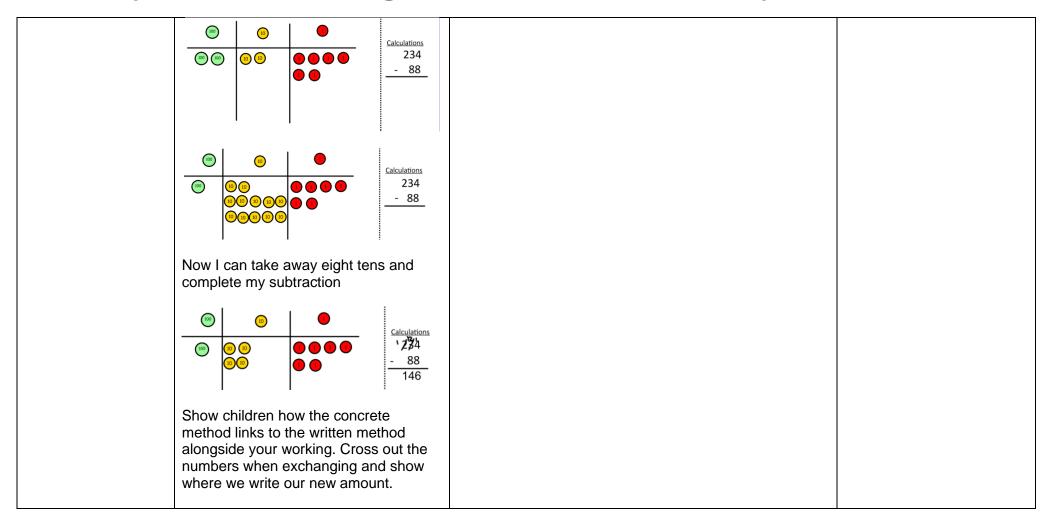


Children can start their formal written method by partitioning the number into clear place value columns.

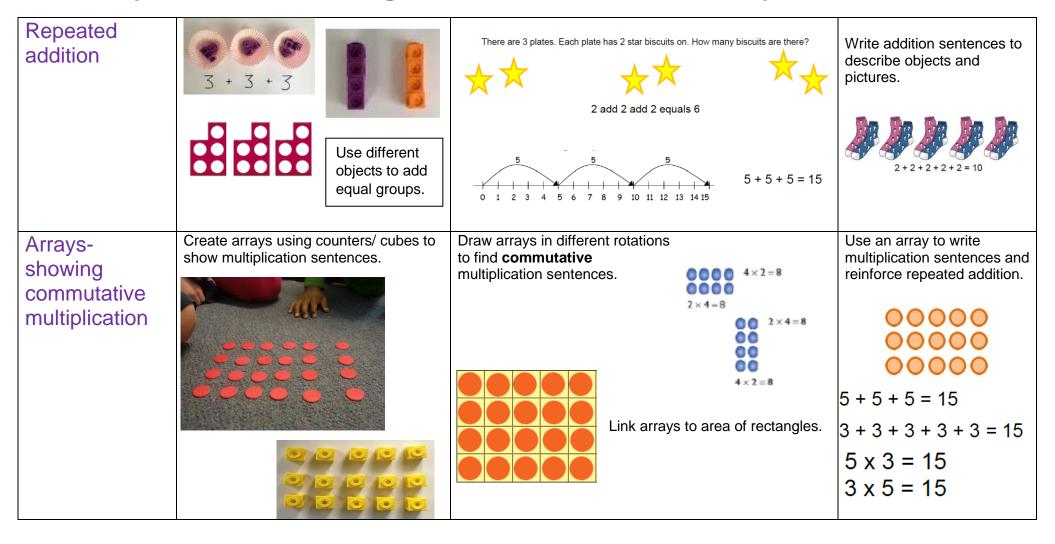


Moving forward the children use a more compact method.

This will lead to an understanding of subtracting any number including decimals.

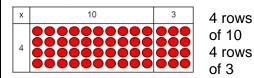


Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.	Draw pictures to show how to double a number. Double 4 is 8	16 10 6 x2 20 12 Partition a number and then double each part before
Counting in multiples	4×2=8	2012 2013 2013 2013 5 10 15 20 25 30	recombining it back together. Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
	Count in multiples supported by concrete objects in equal groups.	Use a number line or pictures to continue support in counting in multiples.	

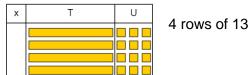


Grid Method

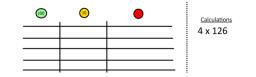
Show the link with arrays to first introduce the grid method.



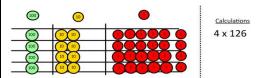
Move on to using Base 10 to move towards a more compact method.



Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



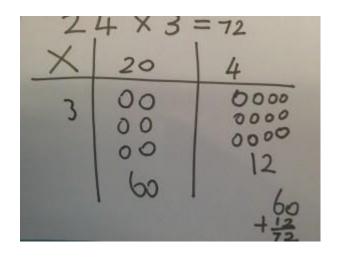
Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.

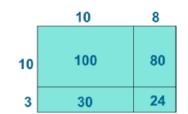


Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

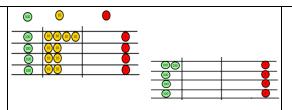
×	30	5
7	210	35

$$210 + 35 = 245$$

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.



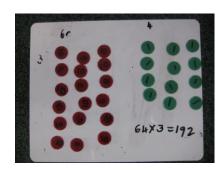
Х	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16



Then you have your answer.

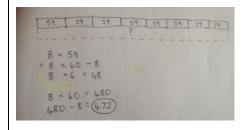
Column multiplication

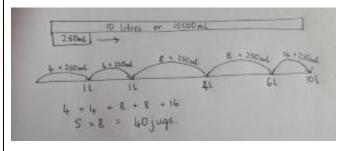
Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.





Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

```
32

x 24

8 (4 x 2)

120 (4 x 30)

40 (20 x 2)

600 (20 x 30)

768
```

This moves to the more compact method.

	1342
	X 18
	10736
	13420
	24156

Division

Objective and	Concrete	Pictorial	Abstract
Strategies			
Sharing objects into groups		Children use pictures or shapes to share quantities.	Share 9 buns between three people. $9 \div 3 = 3$
	I have 10 cubes, can you share them equally in 2 groups?	$8 \div 2 = 4$	
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use a number line to show jumps in groups. The number of jumps equals the number of groups. 0 1 2 3 4 5 6 7 8 9 10 11 12	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
	0 5 10 15 20 25 30 35	Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be each 20 within group. $20 \div 5 = ?$ $5 \times ? = 20$	

